**Johnny’s note: this document uses a number of different ways to build a confidence interval, including CI, group.CI, groupwiseMean, and other similar functions. I just use t.test().**

**Confidence Intervals**

**Packages used in this chapter**

The packages used in this chapter include:

•  Rmisc

•  DescTools

•  plyr

•  boot

•  rcompanion

**Understanding confidence intervals**

Confidence intervals are used to indicate how accurate a calculated statistic is likely to be.  Confidence intervals can be calculated for a variety of statistics, such as the mean, median, or slope of a linear regression.  This chapter will focus on confidences intervals for means.  This book contains a separate chapter, *Confidence Intervals for Medians*, which addresses confidence intervals for medians.  There is also a chapter *Confidence Intervals for Proportions* in this book.

The Statistics Learning Center video in the *Required Readings* below gives a good explanation of the meaning of confidence intervals.

***Populations and samples***

Most of the statistics we use assume we are analyzing a sample which we are using to represent a larger population.  If extension educators want to know about the caloric intake of 7th graders, they would be hard-pressed to get the resources to have every 7th grader in the U.S. keep a food diary.  Instead they might collect data from one or two classrooms, and then treat the data *sample* as if it represents a larger *population* of students.

The mean caloric intake could be calculated for this sample, but this mean will not be exactly the same as the mean for the larger population.  If we collect a large sample and the values aren’t too variable, then the sample mean should be close to the population mean.  But if we have few observations, or the values are highly variable, we are less confident our sample mean is close to the population mean.

We will use confidence intervals to give a sense of this confidence.

It’s best not to overthink the discussion on populations and samples.  We aren’t necessarily actually extending our statistics to a larger population.  That is, we shouldn’t think our measurements from two classrooms are actually indicative of the whole country.  There are likely many factors that would change the result school to school and region to region.  But even if we are thinking about just the 7th graders in just these two classrooms, most of our statistics will still be based on the assumption that there is a larger population of 7th graders, and we are sampling just a subset.

***Statistics and parameters***

When we calculate the sample mean, the result is a *statistic*.  It’s an estimate of the population mean, but our calculated sample mean would vary depending on our sample.  In theory, there is a mean for the population of interest, and we consider this population mean a *parameter*.  Our goal in calculating the sample mean is estimating the population parameter.

***Point estimates and confidence intervals***

Our sample mean is a *point estimate* for the population parameter.  A point estimate is a useful approximation for the parameter, but considering the confidence interval for the estimate gives us more information.

As a definition of confidence intervals, if we were to sample the same population many times and calculated a sample mean and a 95% confidence interval each time, then 95% of those intervals would contain the actual population mean.

If this definition of confidence intervals doesn’t make much intuitive sense to you at this point, don’t worry about it.  Working through some of the examples in this book will help you understand their usefulness.

One use of confidence intervals is to give a sense of how accurate our calculated statistic is relative to the population parameter.

An example

Imagine we have a rule of thumb that we consider a town with a mean household income of greater than $100,000 to be high-income.

For Town A we sample some households, and calculate the mean household income and the 95% confidence interval for this statistic.  The mean is $125,000, but the data are quiet variable, and the 95% confidence interval is from $75,000 to $175,000.  In this case, we don’t have much confidence that Town A is actually a high-income town.  The point estimate for the population mean is greater than $100,000, but the confidence interval extends considerably lower than this threshold.

For Town B, we also get a mean of $125,000, so the point estimate is the same as for Town A.  But the 95% confidence interval is from $105,000 to $145,000.  Here, we have some confidence that Town B is actually a high-income town, because the whole 95% confidence interval lies higher than the $100,000 threshold.

***Confidence intervals as an alternative to some tests***

Most of the statistical tests in this book will calculate a probability (*p*-value) of the likelihood of data and draw a conclusion from this *p*-value.  John McDonald, in the *Optional Readings* below, describes how confidence intervals can be used as an alternative approach.

For example, if we want to compare the means of two groups to see if they are statistically different, we will use a *t*-test, or similar test, calculate a *p*-value, and draw a conclusion.  An alternative approach would be to construct 95% or 99% confidence intervals about the mean for each group.  If the confidence intervals of the two means don’t overlap, we are justified in calling them statistically different.

Likewise, if the 95% confidence interval for some statistic includes zero, we can conclude that the statistic is not significantly different from zero.  
  
As a technical note, non-overlapping confidence intervals for means do not equate exactly to a *t*-test with a *p*-value of 0.05.  They are different methods to assess similar questions.  The article by Cumming and Finch in the “References” section gives more details on the relationship between overlapping confidence intervals and *p*-values from statistical tests.

**Example for confidence intervals**

For this example, extension educators had students wear pedometers to count their number of steps over the course of a day.  The following data are the result.  *Rating* is the rating each student gave about the usefulness of the program, on a 1-to-10 scale.

Input = ("  
Student  Sex     Teacher  Steps  Rating  
a        female  Catbus    8000   7  
b        female  Catbus    9000  10  
c        female  Catbus   10000   9  
d        female  Catbus    7000   5  
e        female  Catbus    6000   4  
f        female  Catbus    8000   8  
g        male    Catbus    7000   6  
h        male    Catbus    5000   5  
i        male    Catbus    9000  10  
j        male    Catbus    7000   8  
k        female  Satsuki   8000   7  
l        female  Satsuki   9000   8  
m        female  Satsuki   9000   8  
n        female  Satsuki   8000   9  
o        male    Satsuki   6000   5  
p        male    Satsuki   8000   9  
q        male    Satsuki   7000   6  
r        female  Totoro   10000  10  
s        female  Totoro    9000  10  
t        female  Totoro    8000   8  
u        female  Totoro    8000   7  
v        female  Totoro    6000   7  
w        male    Totoro    6000   8  
x        male    Totoro    8000  10  
y        male    Totoro    7000   7  
z        male    Totoro    7000   7  
")  
  
Data = read.table(textConnection(Input),header=TRUE)  
  
  
###  Check the data frame  
  
Data  
  
str(Data)  
  
summary(Data)  
  
  
### Remove unnecessary objects  
  
rm(Input)

***Recommended procedures for confidence intervals for means***

Confidence intervals for means can be calculated by various methods.

The *traditional* method is the most commonly encountered, and is appropriate for normally distributed data or with large sample sizes.  It produces an interval that is symmetric about the mean.

For skewed data, confidence intervals by *bootstrapping* may be more reliable.

For routine use, I recommend using bootstrapped confidence intervals, particularly the *BCa* or *percentile* methods.  For further discussion, see below *Optional Analyses: confidence intervals for the mean by bootstrapping*.

**groupwiseMean*function for grouped and ungrouped data***

The *groupwiseMean* function in the rcompanion package can produce confidence intervals both by traditional and bootstrap methods, for grouped and ungrouped data.

The data must be housed in a data frame.  By default, the function reports confidence intervals by the traditional method.

In the *groupwiseMean* function, the measurement and grouping variables can be indicated with formula notation, with the measurement variable on the left side of the tilde (~), and grouping variables on the right.

The confidence level is indicated by, e.g., the *conf = 0.95* argument.  The *digits* option indicates the number of significant digits to which the output is rounded.  Note that in the output, the means and other statistics are rounded to 3 significant figures.

Ungrouped data

Ungrouped data is indicated with a *1* on the right side of the formula, or the *group = NULL* argument.

library(rcompanion)  
  
groupwiseMean(Steps ~ 1,  
              data   = Data,  
              conf   = 0.95,  
              digits = 3)

One-way data

library(rcompanion)  
  
groupwiseMean(Steps ~ Sex,  
              data   = Data,  
              conf   = 0.95,  
              digits = 3)

Two-way data

library(rcompanion)  
  
groupwiseMean(Steps ~ Teacher + Sex,  
              data = Data,  
              conf = 0.95,  
              digits = 3)

Bootstrapped means by group

In the *groupwiseMean* function, the type of confidence interval is requested by setting certain options to *TRUE*.  These options are *traditional*, *normal*, *basic*, *percentile* and *bca*.  The *boot* option reports an optional statistic, the mean by bootstrap.  The *R* option indicates the number of iterations to calculate each bootstrap statistic.

library(rcompanion)  
  
groupwiseMean(Steps ~ Sex,  
              data   = Data,  
              conf   = 0.95,  
              digits = 3,  
              R      = 10000,  
              boot        = TRUE,  
              traditional = FALSE,  
              normal      = FALSE,  
              basic       = FALSE,  
              percentile  = FALSE,  
              bca         = TRUE)

library(rcompanion)  
  
groupwiseMean(Steps ~ Teacher + Sex,  
              data   = Data,  
              conf   = 0.95,  
              digits = 3,  
              R      = 10000,  
              boot        = TRUE,  
              traditional = FALSE,  
              normal      = FALSE,  
              basic       = FALSE,  
              percentile  = FALSE,  
              bca         = TRUE)